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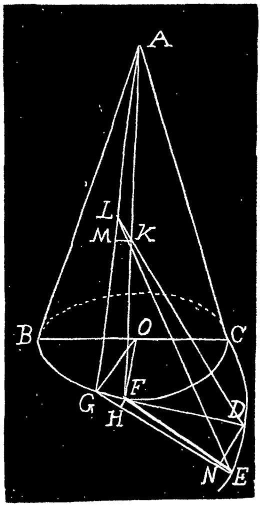
Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let h=height, r=radius of base, $l=\sqrt{(h^2+r^2)}$=slant height, n=number of spirals ; D, E, two consecutive points in the duck's path ; K, L, corresponding consecutive points of tangency of string to cone ; F, G, corresponding consecutive points in base of cone.



Let $CD=s$, $FK=x$. $\therefore$ $DE=ds$, $ML=dx$.

$ds^2=DN^2+NE^2 \ldots\ldots\ldots\ldots\ldots\ldots (1).$

From the similar triangles GOF and DFE,

$r : FG=FD : DN.$ $\therefore$ $DN=(FD.FG)/r \ldots\ldots\ldots (2).$

From the triangles LMK and KFD,

$dx : MK=x : DF.$ $\therefore$ $FD=(x.MK)/dx \ldots\ldots (3).$

$MK : (l-x)=GF : l.$ $\therefore$ $MK=[(l-x).GF]/l \ldots\ldots (4).$

But $CF : x=2\pi rn : l.$

$\therefore$ $GF : dx=2\pi rn : l.$ $\therefore$ $GF=(2\pi rn.dx)/l \ldots\ldots (5).$

(5) in (4) gives, $MK=[2\pi rn(l-x)dx]/l^2 \ldots\ldots (6).$

(6) in (3) gives, $FD=[2\pi rnx(l-x)]/l^2 \ldots\ldots (7).$

(7) and (5) in (2) gives $PN=[4\pi^2r^2n^2(l-x)xdx]/rl^3 \ldots (8).$

The increment of FD is $(GH+NE)=FG+NE)$.

$\therefore$ from (7) $d(FD)=[2\pi rn(l-2x)dx]/l^2=(FG+NE) . (9).$

(5) in (9) gives, $NE=[-4\pi rnxdx]/l^2 \ldots\ldots (10).$

(8) and (10) in (1) gives,

$$ds=\frac{4\pi rnx}{l^2}\sqrt{1+\frac{\pi^2n^2}{l^2}(l-x)^2}dx. \quad \therefore s=\frac{4\pi rn}{l^2}\int_o^l\sqrt{1+\frac{\pi^2n^2}{l^2}(l-x)^2}xdx,$$

$$=(4r/3\pi n)+(2r/3)[\pi n-(2/\pi n)]\sqrt{1+\pi^2n^2}+2r\log(\pi n+\sqrt{1+\pi^2n^2}).$$

In the problem $r=1$, $n=100$.

$$\therefore s=(1/75\pi)+\tfrac{2}{3}[100\pi-(1/50\pi)]\sqrt{1+10000\pi^2}+2\log(100\pi+\sqrt{1+10000\pi^2}).$$

$\therefore$ $s=68948.771$ feet.

66. **Proposed by J. K. ELLWOOD, A. M., Principal of the Colfax School, Pittsburg, Pa.**

Around the top of a conical frustum,—base 5 feet, top 1 foot, altitude 100 feet,—is wound a rope 100 feet long and 1 inch thick. It is unwound by a hawk flying in one plane. How far does Mr. Hawk fly ?

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Instead of measuring the thickness of the rope on the slant height, in this solution it is measured on the altitude, the difference being for the whole winding not above .007 of an inch. Measuring from middle of rope to middle of rope, the first coil is $\frac{1}{2}$ inch below top of frustum, second coil $\frac{3}{2}$ inches, third $\frac{5}{2}$ inches, etc. Let x=height of cone.

$\therefore$ $x : \frac{5}{2}=x-100 : \frac{1}{2}$, $\therefore$ $x=125$.

$125-100=25$ feet, $=300$ inches.

$\therefore 300 : 6 = 300\frac{1}{2} : 6\frac{1}{100}$,

$300 : 6 = 301\frac{1}{2} : 6\frac{3}{100}$,

$300 : 6 = 302\frac{1}{2} : 6\frac{5}{100}$.

. . . .

Let n=number of turns.

$\therefore 2\pi(6\frac{1}{100}+6\frac{3}{100}+6\frac{5}{100}+\text{to } n \text{ terms})=1200$ inches.

$\therefore 2\pi(6+6+6+\ldots\ldots)+2\pi(\frac{1}{100}+\frac{3}{100}+\frac{5}{100}+\ldots\ldots)=1200$.

$\therefore 12\pi n+\pi n^2/1200=100$. $\therefore n=31.03$ coils. From the previous problem,

$$s=\frac{4\pi rm}{l^2}\int_{25}^{27\frac{7}{12}}\sqrt{1+\frac{\pi^2 m^2}{l^2}(l-x)^2}.xdx, \text{ where } m=331,$$

the number of coils of rope to top of cone beginning with the bottom coil, and $l=331$ inches the height$=27\frac{7}{12}$ feet ; also let 25 feet$=l'$, $r=\frac{331}{600}$ feet=radius of base of cone at bottom of rope.

$$\therefore s=\frac{4\pi rm}{l^2}\int_{l'}^{l}\sqrt{1+\frac{\pi^2 m^2}{l^2}(l-x)^2}.xdx.$$

$$\therefore s=\frac{4r}{3\pi m}+\frac{2r}{3l^3\pi m}[(l^2+ll'-2l'^2)\pi^2 m^2-2l^2]\sqrt{l^2+(l-l')^2\pi^2 m^2}$$

$$+2r\log\left(\frac{(l-l')\pi m+\sqrt{l^2+(l-l')^2\pi^2 m^2}}{l}\right).$$

$$s=(1/450\pi)+(1/900\pi)(28861\pi^2-2)\sqrt{1+961\pi^2}+\tfrac{331}{300}\log(31\pi+\sqrt{1+961\pi^2}).$$

$s=9817.69235$ feet.

In this solution the rope is unwound from the bottom.

MECHANICS.

55. Proposed by **ALFRED HUME, C. E., D. Sc.,** Professor of Mathematics, University of Mississippi, University P. O., Miss.

Three equal heavy spheres, each of weight W, are placed on a rough ground just not touching each other. A fourth sphere of weight nW is placed on the top touching all three. Show that there is equilibrium if the coefficient of friction between two spheres is greater than $\tan\frac{1}{2}\alpha$, and that between a sphere and the ground is greater than $n\tan\frac{1}{2}\alpha/(n+3)$, where α is the inclination to the vertical of the straight line joining the centers of the upper and one lower sphere.

Solution by the PROPOSER.

I.

In what different ways may motion occur, if the friction be such as to permit motion ?

So long as the lower spheres maintain their positions, the upper will not move.